Discrete Element Approach to Flutter of Skew Panels with In-Plane Forces under Yawed Supersonic Flow

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The flutter problem of skew panels, with inplane forces under yawed supersonic flow, has been considered in this paper through the application of matrix displacement methods. Necessary kinematically consistent aerodynamic influence coefficient matrices (AIC), which are applicable to yawed supersonic flows, have been developed and are employed in the formulation of the dynamic equations of motion. The results have also been compared with the other available results and the agreement is found satisfactory. This approach makes it possible to tackle flutter problems of panels with practically any boundary condition including cut-outs, if any, and subjected to thermal or other midplane forces.

Nomenclature

$\mathbf{a}$ = Boolean matrix describing topological connections of the discrete elements
$\mathbf{a}_e$ = transformation matrix describing the relation between natural and kinematic modes[5]
a,b = length of the element along X_1 and Y_1, respectively
$\mathbf{A}_0,\mathbf{A}$ = elemental and system aerodynamic damping matrix, respectively
$\mathbf{B}$ = assembled aerodynamic stiffness matrix
$\mathbf{B}_{01},\mathbf{B}_{02}$ = elemental aerodynamic stiffness matrices
C_0 = $ql^3/2\beta D$
C_1 = $(M^2 - 2)C_0/\beta^2$
C_2 = $(\beta/4)(\rho m/\rho a)(h/l)C_0$
D = plate rigidity
h = thickness of panel
$\mathbf{K}_E$ = elastic stiffness matrix
$\mathbf{K}_G,\tilde{\mathbf{K}}_G$ = geometric stiffness matrix
k_1,k_2 = constants [Eq. (9)]
l = reference length (chord)
M = Mach number
$\mathbf{M}$ = inertia matrix
p = intensity of aerodynamic pressure
$\mathbf{q}$ = dynamic pressure $(\rho_a U^2/2)$
$\tilde{\mathbf{q}}$ = $\mathbf{q}e^{\bar{\lambda}t}$
Q = aerodynamic parameter $4C_0/\pi^4$
r_x = nondimensional magnitude of edge loading $(\sigma_0 hl^2/\pi^2 D)$
s = span of the panel
t = dimensional time
U = freestream velocity
$\tilde{W}$ = work done
W = vertical displacement
w = nondimensional displacement (W/l)
$\bar{x},\bar{y},\bar{z}$ = rectangular coordinates
x,y,z = yawed rectangular coordinates
x_1,y_1,z = oblique coordinates
α = damping coefficient
β = $(M^2 - 1)^{1/2}$
δ = angle of yaw
Λ = complementary sweep angle
ψ = angle of sweep
ω = circular frequency
$\omega_{xx},\omega_{yy},\omega_{xy}$ = factors defining the intensity of midplane stresses
$\bar{\omega}$ = transformation matrix function
Ω = $\bar{\omega}\mathbf{a}_e$
ξ,η = nondimensional oblique coordinates of the element
λ,μ = nondimensional lengths of the element
$\bar{\lambda}$ = $(\alpha l/U + i\omega l/U)$
$\bar{\mu}$ = complex eigenvalue
ζ = $(C_2)^{1/2}\bar{\lambda}$
ρ_a = density of air
ρ_m = density of panel material
$\boldsymbol{\rho}$ = column vector of kinematic modes

Superscripts

$(\dot{\ })$ = differentiation with respect to physical time t
t = transposed matrix

Subscripts

ξ = differentiation with respect to ξ
η = differentiation with respect to η

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1. Introduction

THE problem of panel flutter with in-plane loads is very practical since the aerospace structures are often subjected to thermal loadings and loads due to edge constraints which lead to stressed skins. Hedgepeth[1] has treated rectangular panels under spanwise and chordwise normal edge forces. Eisley and Luessen[2] have considered the effect of combined shear and normal forces on parallelogramic plates. In general, they have found that compressive forces reduce the critical dynamical load and in particular the shear load is a significant factor in the panel stability.

There exists a large number of recent references which consider the mechanism of panel flutter under various conditions.[1–3,6–9,12] Classical methods employ small deflection thin plate theory and two-dimensional quasi-static aerodynamic theory. In general the method of solution uses double Fourier sine series for simply supported panels and other modified functions for different boundary conditions. In effect the entire algebra has to be performed for each case. To overcome this difficulty finite-element approaches have been proposed in Refs. 4, 10, and 11. These methods accommodate any boundary conditions very conveniently by appropriate computer instructions in a general program.

In Ref. 4, for example, a finite-element approach has been used for the study of panel flutter problems. In the same work a method has also been proposed for deriving the aerodynamic influence coefficient matrices which are kinematically consistent with the inertia and stiffness matrices. Though the possibility of its application to skew panels and for panels with in-plane stresses was mentioned in Ref. 4, the theoretical development and the numerical results were restricted to rectangular panels with straight supersonic flow. The use

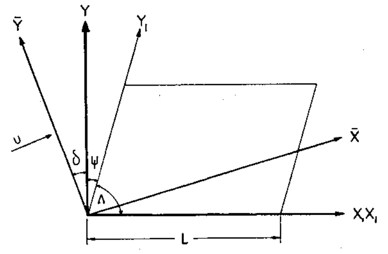

Fig. 1 Panel and coordinate system.

of yawed flow can very conveniently be accounted for by means of a coordinate transformation in the derivation of the necessary aerodynamic influence coefficient matrices. Hence in the present work, the scope of the matrix displacement method for the study of panel flutter is further extended to cover the case of yawed flow and is applied to skew panels subjected to in-plane loads.

2. Basic Equations of Motion

The dynamic equations for a nonconservative system, under the influence of elastic, in-plane, inertial, and aerodynamic forces may be written as

$$[\mathbf{K}_E - (\pi^2/16)r_x\overline{\mathbf{K}}_G]\tilde{q} + C_0[\mathbf{B}]\tilde{q} + C_1[A]\dot{\tilde{q}} + C_2[\mathbf{M}]\ddot{\tilde{q}} = 0 \tag{1}$$

The elastic stiffness ($\mathbf{K}_E$), geometric stiffness ($\mathbf{K}_G$), and inertia ($\mathbf{M}$) matrices can be obtained from Ref. 5. The derivation of aerodynamic influence coefficient matrices ($\mathbf{A}$ and $\mathbf{B}$) is briefly described below and is applicable to yawed supersonic flow.

a. Derivation of Aerodynamic Influence Coefficient Matrices

Assuming the quasi-steady supersonic theory, the intensity of aerodynamic pressure at any point (x,y) may be repre-

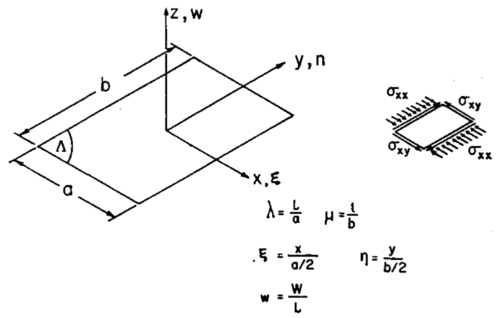

Fig. 2 Typical discrete element.

sented as

$$p(x,y,t) = \frac{\rho_a U^2}{\beta}\left[\frac{\partial w}{\partial \bar{x}} + \frac{\beta^2 - 1}{\beta^2}\left(\frac{1}{U}\right)\frac{\partial w}{\partial t}\right] \tag{2}$$

Referring to the axes system (Fig. 1), one can derive the necessary coordinate transformation as

$$\begin{bmatrix}\bar{x}\\ \bar{y}\end{bmatrix} = \begin{bmatrix}\cos\delta & \sin\delta\\ -\sin\delta & \cos\delta\end{bmatrix}\begin{bmatrix}1 & \sin\psi\\ 0 & \cos\psi\end{bmatrix}\begin{bmatrix}x_1\\ y_1\end{bmatrix}$$

or

$$\begin{bmatrix}x_1\\ y_1\end{bmatrix} = \begin{bmatrix}1 & -\tan\psi\\ 0 & \sec\psi\end{bmatrix}\begin{bmatrix}\cos\delta & -\sin\delta\\ \sin\delta & \cos\delta\end{bmatrix}\begin{bmatrix}\bar{x}\\ \bar{y}\end{bmatrix} \tag{3}$$

Hence the derivative

$$\partial w/\partial \bar{x} = (\partial w/\partial x_1)\partial x_1/\partial \bar{x} + (\partial w/\partial y_1)\partial y_1/\partial \bar{x} \tag{4}$$

where

$$\partial x_1/\partial \bar{x} = \cos\delta - \sin\delta\cdot\tan\psi = k_1 \tag{5}$$

and

$$\partial y_1/\partial \bar{x} = \sin\delta\cdot\sec\psi = k_2 \tag{6}$$

Table 1 Matrix β_{01} − AIC

$$\bar{\beta}_{01} = \begin{bmatrix}
0 & 0 & \frac{1}{120} & 0 & 0 & \frac{1}{60} & 0 & 0 & 0 & 0 & \frac{1}{24} & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{60} & 0 & 0 & 0 & 0 & \frac{1}{24} & 0\\
-\frac{1}{120} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{420} & \frac{1}{60} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{17}{4200} & \frac{3}{700} & \frac{1}{30} & 0 & 0 & 0\\
-\frac{1}{60} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \frac{-17}{4200} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \frac{-3}{700} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -\frac{1}{30} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{12} & 0 & 0 & 0 & 0 & \frac{1}{4} & 0\\
-\frac{1}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{120} & -\frac{1}{12} & 0 & 0 & 0
\end{bmatrix}$$

$$\mathbf{d}_1 = \begin{bmatrix}\frac{1}{\lambda} & \frac{1}{\mu} & \frac{1}{\lambda} & \frac{1}{\mu} & \frac{1}{\lambda} & \frac{1}{\mu} & \frac{1}{\lambda} & \frac{1}{\mu} & 1 & 1 & \frac{1}{\lambda} & \frac{1}{\mu}\end{bmatrix}$$

$$\beta_{01} = \frac{1}{\lambda\mu}\,\mathbf{d}_1\bar{\beta}_{01}\mathbf{d}_2 \qquad \mathbf{d}_2 = \begin{bmatrix}1 & 1 & 1 & 1 & 1 & \frac{\lambda}{\mu} & 1 & \frac{\lambda}{\mu} & \lambda & 1 & 1 & 1\end{bmatrix}$$

Table 2 Matrix β_{02} — AIC

$$
\bar{\beta}_{02} = \begin{bmatrix}
0 & 0 & 0 & 0 & +\frac{1}{60} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{24} \\
0 & 0 & 0 & +\frac{1}{120} & +\frac{1}{60} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{24} \\
0 & 0 & 0 & 0 & 0 & 0 & +\frac{1}{420} & 0 & +\frac{1}{60} & 0 & 0 & 0 \\
0 & -\frac{1}{120} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{60} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & +\frac{3}{700} & +\frac{17}{4200} & +\frac{1}{30} & & & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{3}{700} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{17}{4200} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{30} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & +\frac{1}{12} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} \\
0 & 0 & 0 & 0 & 0 & 0 & +\frac{1}{120} & 0 & +\frac{1}{12} & 0 & 0 & 0 \\
0 & +\frac{1}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$\beta_{02} = \frac{1}{\lambda\mu}\mathbf{d}_1\bar{\beta}_{02}\mathbf{d}_3 \qquad \mathbf{d}_3 = \left\lceil 1 \;\; 1 \;\; 1 \;\; 1 \;\; \frac{\mu}{\lambda} \;\; 1 \;\; \frac{\mu}{\lambda} \;\; 1 \;\; \mu \;\; 1 \;\; 1 \;\; 1 \right\rfloor$$

Then Eq. (2) can be written as

$$p(x,y,t) = \frac{\rho_a U^2}{\beta}\left[k_1\frac{\partial w}{\partial x_1} + k_2\frac{\partial w}{\partial y_1} + \frac{\beta^2 - 1}{\beta^2}\left(\frac{1}{U}\right)\frac{\partial w}{\partial t}\right] \tag{7}$$

Now introducing the nondimensional parameters, (Fig. 2)

$$\xi = x_1/(a/2),\ \eta = y_1/(b/2),\ \lambda = l/a,\ \mu = l/b$$

and $w = W/l$ where l is any reference length (say chord), and further expressing the nondimensional deflection w in terms of kinematic modes (see Ref. 5) as

$$w = \mathbf{\Omega}\boldsymbol{\varrho} \tag{8}$$

where $\mathbf{\Omega}$ is a transformation matrix function.[5] One can rewrite the Eq. (7) as

$$p(x,y,t) = (\rho_a U^2/\beta)\{k_1\mathbf{\Omega}_\xi\boldsymbol{\varrho} + k_2\mathbf{\Omega}_\eta\boldsymbol{\varrho} + [(\beta^2 - 1)/\beta^2] \times (l/U)\mathbf{\Omega}\dot{\boldsymbol{\varrho}}\} \tag{9}$$

In which

$$\mathbf{\Omega}_\xi = 2\lambda\partial\mathbf{\Omega}/\partial\xi$$

and

$$\mathbf{\Omega}_\eta = 2\mu\partial\mathbf{\Omega}/\partial\eta$$

The virtual work done by the aerodynamic forces may then be written as

$$\delta\bar{W} = \frac{1}{2}\left(\frac{\sin\Lambda\cdot l^3}{4\lambda\mu}\right)\int_{-1}^{1}\int_{-1}^{1}\delta w^t p\, d\xi d\eta \tag{10}$$

Then from the principle of virtual work the aerodynamic forces acting in the direction of the kinematic description may be obtained as

$$\mathbf{p} = \frac{ql^3\sin\Lambda}{2\beta}\left[k_1\mathbf{B}_{01}\boldsymbol{\varrho} + k_2\mathbf{B}_{02}\boldsymbol{\varrho} + \frac{\beta^2 - 1}{\beta^2}\left(\frac{l}{U}\right)\mathbf{A}_0\dot{\boldsymbol{\varrho}}\right] \tag{11}$$

where $\mathbf{B}_{01}$, $\mathbf{B}_{02}$, and $\mathbf{A}_0$ are the required AIC matrices for the element and are given by

$$\mathbf{B}_{01} = \frac{1}{\lambda\mu}\int_{-1}^{1}\int_{-1}^{1}\mathbf{\Omega}^t\mathbf{\Omega}_\xi\, d\xi d\eta = \mathbf{a}_e{}^t\beta_{01}\mathbf{a}_e$$

$$\mathbf{B}_{02} = \frac{1}{\lambda\mu}\int_{-1}^{1}\int_{-1}^{1}\mathbf{\Omega}^t\mathbf{\Omega}_\eta\, d\xi d\eta = \mathbf{a}_e\beta_{02}\mathbf{a}_e$$

and

$$\mathbf{A}_0 = \frac{1}{\lambda\mu}\int_{-1}^{1}\int_{-1}^{1}\mathbf{\Omega}^t\mathbf{\Omega}\, d\xi d\eta = \mathbf{a}_e{}^t A_0\mathbf{a}_e \tag{12}$$

in which $\mathbf{a}_e$ is a transformation matrix relating the natural and the system kinematic mode vectors as described by Argyris.[5]

The elemental matrices β_{01} and β_{02} are given in Tables 1 and 2.

Knowing the AIC matrices for the elements it is straight forward to assemble them with the help of Boolean operations to obtain the required matrices for the panel system, i.e.,

$$\mathbf{B} = \mathbf{a}^t(k_1\mathbf{B}_{01} + k_2\mathbf{B}_{02})\mathbf{a},\ \mathbf{A} = \mathbf{a}^t\mathbf{A}_0\mathbf{a} \tag{13}$$

b. Midplane Forces

The effect of midplane forces can very conveniently be included in the analysis using matrix methods by the concept of geometric stiffness ($\mathbf{K}_G$) introduced by Argyris.[5] The derivation of kinematically consistent geometric stiffness matrix ($\mathbf{K}_G$) is similar in principle to that of elastic stiffness ($\mathbf{K}_E$) and inertia ($\mathbf{M}$) matrices. For the present analysis, $\mathbf{K}_G$ for a parallelogramic plate is extracted from Ref. 5 and reads as

$$\tilde{\mathbf{K}}_G = [(\sigma_0 h)l^2\sin\Lambda/16][\omega_{xx}\lambda^2\mathbf{B}_1 + \omega_{yy}\mu^2\mathbf{B}_2 - \omega_{xy}\lambda\mu\mathbf{B}_3] \tag{14}$$

where $\sigma_0 h$ shows the magnitude of the edge load per unit length and, ω_{xx}, ω_{yy}, and ω_{xy} refer to the description of the midplane loading, i.e.,

$$\sigma_{xx}h = (\sigma_0 h)\omega_{xx},\ \sigma_{yy}h = (\sigma_0 h)\omega_{yy},\ \sigma_{xy}h = (\sigma_0 h)\omega_{xy} \tag{15}$$

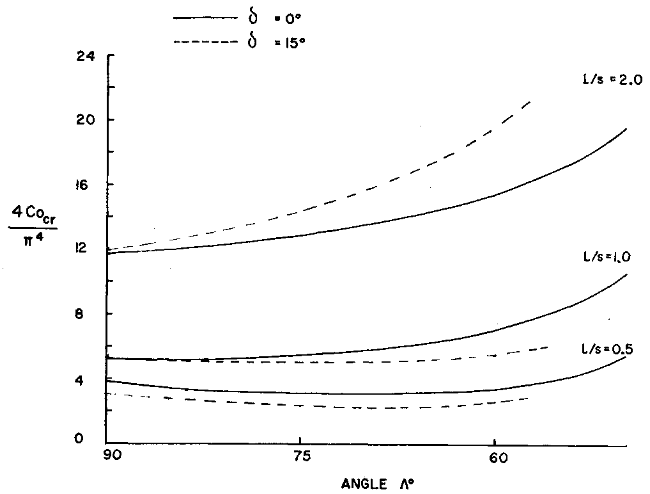

Fig. 3 Variation of critical aerodynamic parameter with Λ.

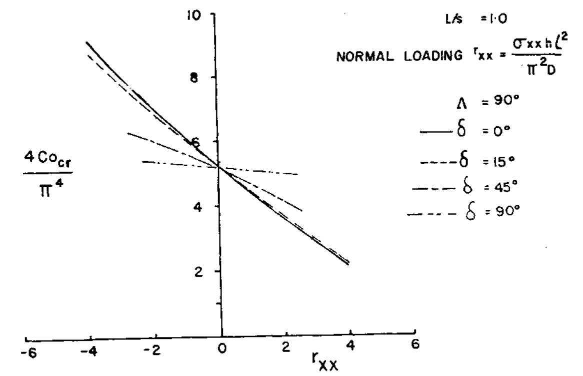

Fig. 4a Variation of critical dynamic pressure with in-plane force r_{xx}, for $\Lambda = 90°$.

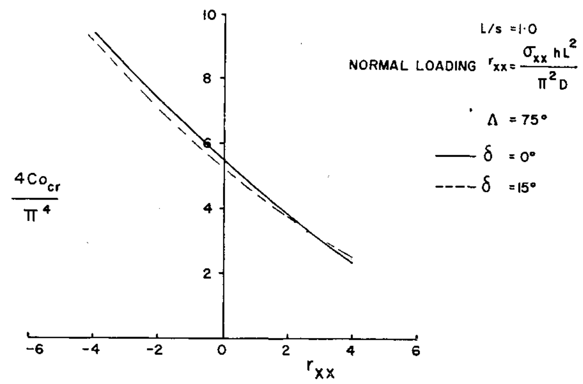

Fig. 4b Variation of critical dynamic pressure with in-plane force r_{xx}, for $\Lambda = 75°$.

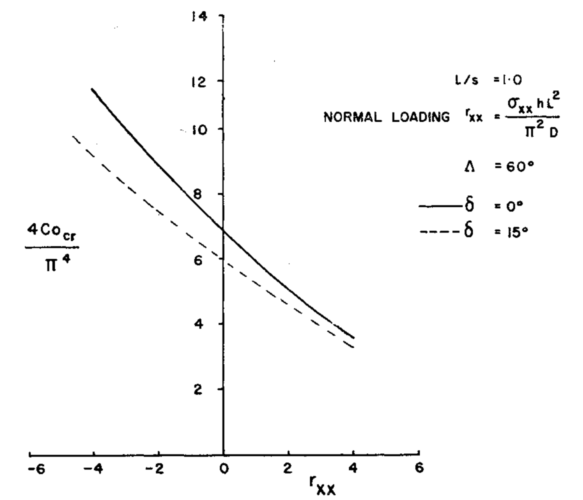

Fig. 4c Variation of critical dynamic pressure with in-plane force r_{xx}, for $\Lambda = 60°$.

In Eq. (1) the term r_x represents $(\sigma_0 hl^2/\pi^2 D)$ and hence the required magnitude of loading can be obtained just by varying r_x for a suitable combination of ω_{xx}, ω_{yy}, and ω_{xy}.

c. Solution of the Equation of Motion

As the aerodynamic damping matrix (**A**) and the inertia matrix (**M**) both depend on the time derivatives of the assumed natural modes, they differ by only a scalar multiplier. Hence, with the assumption $\mathbf{A} = k\mathbf{M}$, the Eq. (1) may be reduced to

$$[\mathbf{K}_E - (\pi^2 r_x/16)K_G + C_0\mathbf{B}]\mathbf{q} + (C_1 k\bar{\lambda} + C_2\bar{\lambda}^2)\mathbf{M}\mathbf{q} = 0$$

or in the conventional eigenvalue form, it can be represented as

$$[\mathbf{K}_E - (\pi^2 r_x/16)\mathbf{K}_G + C_0\mathbf{B}]^{-1}\mathbf{M}\mathbf{q} = \bar{\mu}\mathbf{q} \tag{16}$$

where $\bar{\mu}$ is an eigenvalue.

In the present numerical analysis the effect of aerodynamic damping $(C_1 = 0)$ is neglected and hence

$$\bar{\mu} = -1/C_2\bar{\lambda}^2 = -1/\zeta$$

or

$$\zeta = \zeta_R + i\zeta_I = (C_2)^{1/2}\bar{\lambda}$$

Hence for each value of r_x, the midplane loading parameter, the eigenvalue system can be solved for various values of C_0, the aerodynamic parameter. For $C_0 = 0$ and $r_x = 0$, the eigenvalues correspond to the natural frequencies. As C_0 is increased two sets of eigenvalues approach each other and after a certain value they become complex. The value of C_0 for which the two eigenvalues coalesce gives the critical value, since no damping is considered.

2. Results and Conclusions

This numerical analysis treats simply supported panels idealized by parallelogramic plate elements. The computed results illustrate the effect on the flutter boundaries of

1) Different skew and yaw angles on unloaded panels of side ratios $l/s = 0.5, 1.0, 2.0$;

2) Normal and shear edge loadings on panels of $l/s = 1.0$ for various skew and yaw angles;

3) A number of finite elements in the panel model (for convergence study). All the results presented in Table 3 and Figs. 3–5 are based on a 5×5 element grid system.

Figure 3 shows the variation of critical aerodynamic parameter $Q_{cr} = (4C_0/\pi 4)cr$ with the angle of skew. For zero yaw $(\delta = 0°)$ Q_{cr} increased monotonically with skewing for $l/s = 1.0, 2.0$. But for $l/s = 0.5$, Q_{cr} decreases initially up to $\Lambda = 75°$ and begins to rise. In the same Fig. 3 the yaw angle $(\delta = 15°)$ results in a destabilizing effect for a panel with $l/s = 0.5$ and stabilizing effect for $l/s = 2.0$, at all skew angles considered herein. However, for $l/s = 1.0$ at $\Lambda = 90°$ the stabilizing effect is negligibly small, but tends to destabilizing with skewing. A similar tendency has also been observed in Ref. 3.

Figure 4 shows the influence of normal edge load $r_{xx} = r_x \cdot \omega_{xx}$ on Q_{cr} for a panel of $l/s = 1.0$ and various yaw and skew combinations. Positive sign of r_{xx} signifies compression

Table 3 Some results of simply supported parallelogramic panel flutter analysis

l/s	Λ, deg	δ, deg	r_x $\omega_{xx} = 1$	r_x $\omega_{xy} = 1$	Q_{cr} (present result)	Q_{cr} (Ref. 2)	Q_{cr} (Ref. 3)
0.5	90	0	0	0	3.84	3.9	3.95
	75	0	0	0	3.12		3.06
	60	0	0	0	3.34		3.56
	90	15	0	0	3.0	2.83	2.65
	75	15	0	0	2.28		2.44
	60	15	0	0	2.5		3.63
1.0	90	0	0	0	5.32	5.2	5.2
	75	0	0	0	5.63		5.57
	60	0	0	0	6.86		7.29
	90	15	0	0	5.35	5.22	
	75	15	0	0	5.00		5.47
	60	15	0	0	5.95		7.06
	90	0	+3	0	2.83	2.71	
	90	0	−3	0	8.10	8.12	
	90	0	0	2	5.0	5.0	
	90	0	0	4	4.3	4.17	
	90	0	0	6	3.3	3.12	
2.0	90	0	0	0	11.78	10.8	10.8
	75	0	0	0	12.8		12.7
	60	0	0	0	15.4		17.6
	90	15	0	0	12.20		
	75	15	0	0	15.0		13.10
	60	15	0	0	20.0		18.13

and of r_{xy} signifies shear load applied as to decrease Λ. Figures 4a–4c show that Q_{cr} decreases monotonically with r_{xx} and the influence of yaw is shown as dotted lines. As yaw is increased, the solid line in Fig. 4a rotates continuously until it becomes nearly horizontal. This signifies that the normal edge load across the stream line hardly has any effect on Q_{cr}.[1,12] Influence of cartesian shear load on rhombic panels is presented in Figs. 5a–5c. For $\Lambda = 90$ and $\delta = 0$, Q_{cr} is symmetrical. But for $\delta = 15°$ positive shear which develops compressional along the diagonal nearer to the streamlines has a destabilizing effect while "negative shear" stabilizes the panel. Similar features are seen in Figs. 5b and 5c, for $\Lambda = 75°$ and 60°, respectively.

A summary of results for the purpose of comparison with Refs. 2 and 3 are shown in Table 3. All these results are based on a 5 × 5 grid system. For side ratios $(l/s) = 0.5$, 1.0, the present results are seen to be in good agreement with those from classical methods. But for $(l/s) = 2.0$ the present results deviate considerably from those of Refs. 2 and 3. One main reason for this is that the orientation of the long side of the panel promotes coupling of the modes aerodynamically. In these coupling modes there is more than one-half wave. To represent these modes adequately there must be a sufficient number of elements. In order to achieve convergence the number of elements in the streamwise direction was therefore increased, but in the spanwise direction was limited because of storage considerations. It was observed that the flutter mode involved coalescence of the one-half wave mode in the spanwise direction, and the two-half wave mode in the streamwise direction, thus justifying the number of elements used.

However, it is not possible to conclude whether the converged results in Table 4, in the absence of exact results, are conservative or nonconservative.

Table 4 Convergence study: simply supported panels $l/s = 2.0$

Grid size	Λ, deg	Present Q_{cr} $\delta = 0$	Present Q_{cr} $\delta = 15$	Remarks $\delta = 0$	Remarks $\delta = 15$
4 × 4	90°	15.5	15.8		
5 × 5		11.78	12.2		
7 × 4		12.20	12.8		
8 × 3		11.4	11.7	10.8[a,b]	
4 × 4	75°	18.0	19.0		
5 × 5		12.80	15.0		
7 × 4		13.10	13.2		
8 × 3		12.25	12.8	12.7[b]	13.10[b]
4 × 4	60°	21.2	21.4		
5 × 5		15.4	20.0		
7 × 4		16.0	16.83		
8 × 3		15.9	16.5	17.6[b]	18.13[b]

[a] Ref. 2.
[b] Ref. 3.

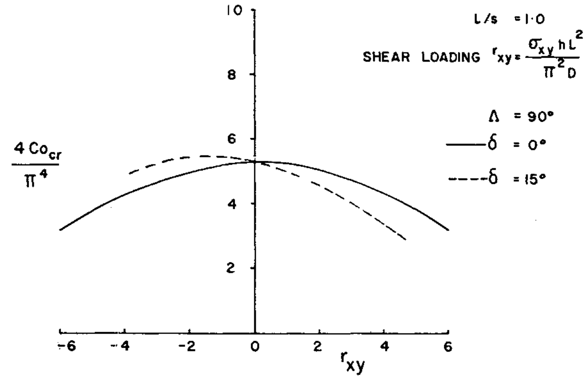

Fig. 5a Variation of critical dynamic pressure with in-plane shear r_{xy}, for $\Lambda = 90°$.

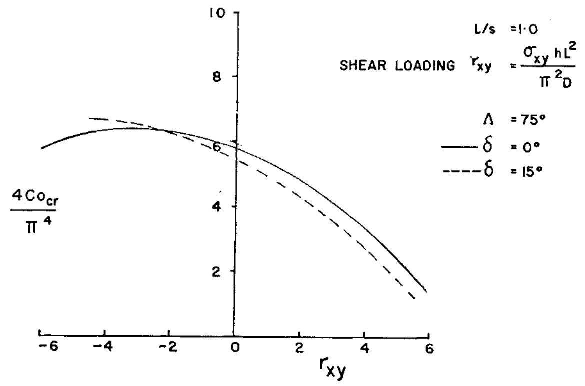

Fig. 5b Variation of critical dynamic pressure with in-plane shear r_{xy}, for $\Lambda = 75°$.

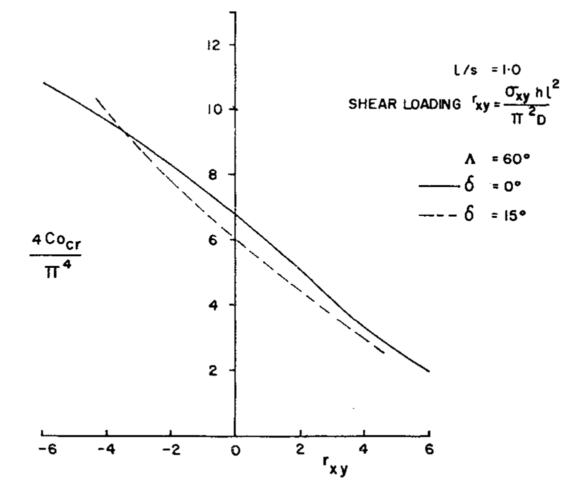

Fig. 5c Variation of critical dynamic pressure with in-plane shear r_{xy}, for $\Lambda = 60°$.

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Comparison of Theory and Experiment for Nonlinear Flutter of Loaded Plates

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The flutter behavior of clamped plates exposed to transverse pressure loadings, or buckled by uniform thermal expansion has been investigated theoretically, and the results compared with existing experimental data. Quasi-steady aerodynamic theory and von Kármán's plate equations are employed. Two sets of in-plane boundary conditions are considered: 1) zero in-plane motion normal to the edges, and 2) zero in-plane stress at the edges. A modal expansion of the transverse deflection is used in conjunction with Galerkin's method to obtain a set of nonlinear ordinary differential equations which are integrated numerically to determine the flutter motion. Good correlation is obtained between experimental and theoretical flutter boundaries for plates exposed to a static pressure differential. The stability boundaries of low aspect ratio plates with zero edge restraint are found to be more sensitive to pressure loads than are those of plates with complete edge restraint. Moreover, comparisons with available experimental data indicate that zero edge restraint is a good assumption for some panel configurations. Finally, it is indicated that fair agreement between theory and experiment can be obtained for buckled plates.

Nomenclature

a = plate length
a_n = modal amplitude
b = plate width
c = speed of sound
d = cavity depth
D $\equiv Eh^3/12(1-\nu^2)$ = plate stiffness
E = Young's modulus
h = plate thickness
I = area moment of inertia
K $\equiv \omega(\rho_m h a^4/D)^{1/2}$ = dimensionless radian frequency
k_x, k_y = in-plane spring constants
M = Mach number
N_x, N_y = in-plane stresses
p = pressure
P $\equiv pa^4/Dh$ = dimensionless pressure
t = time
u = plate in-plane displacement, x direction
U = flow velocity
v = plate in-plane displacement, y direction
w = transverse plate deflection
x,y,z = plate coordinates
α = coefficient of thermal expansion
α_x, α_y = in-plane restraint parameters [see Eq. (15)]
β $\equiv (M^2-1)^{1/2}$
Δp = static pressure differential
ΔP $\equiv \Delta p a^4/D$ = dimensionless static pressure differential
ΔT = temperature differential
Δ_x, Δ_y = in-plane stretchings [see Eq. (5)]
μ $\equiv \rho a/\rho_m h$ = dimensionless flow density
ν = Poisson's ratio
ρ = air density
ρ_m = plate density
τ $\equiv t(D/\rho_m h a^4)^{1/2}$ = dimensionless time

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